Coupling Holes Between Resonant Cavities or Waveguides Evaluated in Terms of Volume Ratios

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Summary—A hole in a common wall is used to provide coupling between two resonant cavities (k = coefficient of coupling) or between two waveguides (x or b = normalized reactance or susceptance) or between cavity and waveguide (p = loading power factor of cavity).

Referring to either side of a thin common wall, the field intensity in the center of a small hole is 1/2 what it would have been at that location on the wall. Between two equal regions, the coupling (k, x or b) by magnetic or electric field is expressed as 1/4 the ratio of the effective volume of the hole over the effective volume of each region. By duality (Booker's principle), the effective volume (related to the polarizability) of an aperture in a thin wall is identified with that of an analogous thin body in a uniform field. For a resonant cavity loaded by coupling to a waveguide, the loading power factor is $p = kx$; this theorem is proved by reference to an equivalent network.

Various cases of coupling by two-dimensional and three-dimensional fields are formulated in terms of area or volume ratios, especially between pillbox resonators (rectangular, circular, or coaxial-circular) and between rectangular waveguides with common side walls or top and bottom walls. The effective area or volume of a small hole in a thin conducting wall is given for various symmetrical shapes, in a magnetic or electric field.

I. Introduction

In MICROWAVE technology, resonant cavities or waveguides may be coupled through a hole in a common wall. The general principles of such coupling are well known (see [3]–[5], [7], [12]).

The present purpose is the presentation of this subject in a unified manner which will emphasize the principles and also will aid in understanding and computing the coupling in a variety of situations. The presentation is quantitative to the extent permitted by some simplifying assumptions.

While most of the subject matter is taken from diverse sources in the literature, the basis for the unified presentation seems to be new. This naturally leads to interesting concepts and relationships that may not have been apparent in the earlier publications. One theorem that is particularly useful enables a simple computation of the loading power factor of a resonant cavity by coupling with a nonreflecting waveguide.

The field coupling or polarizability of any coupling hole will be expressed in terms of an effective volume. The coupled cavitities or waveguides will likewise be evaluated in terms of an effective volume. Then it becomes possible to formulate, in terms of a volume ratio, the coupling coefficient, the normalized reactance or susceptance, or the loading power factor, in any situation where the concept is applicable.

Manuscript received October 7, 1963; revised December 10, 1963. Presented to Wheeler Laboratories Engineering Seminar, March, 1960; also to Long Island Chapter of PGMTT, January, 1961. Presented in part before IEEE-PTGMTT Annual Symposium, Santa Monica, Calif., May, 1963.

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II. Symbols

MKS rationalized units.

E = electric field intensity in space; voltage gradient in resistance sheet (volts/meter).

H = magnetic field intensity in space; current density in resistance sheet (amperes/meter).

V = voltage.

I = current.

R = resistance.

G = conductance.

X = reactance.

B = susceptance.

R_0, G_0 = wave resistance or conductance in a waveguide (based on voltage and power).

f = frequency.

λ = wavelength.

$\lambda/2\pi$ = radianlength.

λ_0 = wavelength in free space.

$\lambda_c = 2a$ = cutoff wavelength in rectangular waveguide.

a, b, c = width, height, length of bounded space.

a = radius of circular cylinder.

a, b = outer and inner radii of coaxial cylinders.

$d = 2r$ = width of aperture; diameter of circle.

r = radius of circle.

r_1, r_2 = major and minor radii of ellipse (disc in uniform field or hole in thin wall).

πr^2 = area of circle.

$\frac{4}{3}\pi r^3$ = volume of sphere.

t = thickness of wall.

l = length of equivalent dipole (for defining polarizability).

A = area.

A_c, A_e, A_m = (see V below).

V = volume.

V_e, V_m = effective volume of electric or magnetic energy in a resonant cavity or waveguide, referred to the field intensity at a point on one wall where coupling hole is to be located.

V_c = effective volume or polarizability of thin body in uniform field, or effective volume of hole in thin wall (see *Note*).

$\frac{1}{4}V_c$ = polarizability of hole in thin wall (Bethe) (see *Note*).

k = coefficient of coupling = ratio of mutual impedance over mean self-impedance of two circuits coupled together, or analogous ratio for two field regions (especially resonators).

$x = X/R_0$ = normalized reactance in waveguide.

$b = B/G_0$ = normalized susceptance in waveguide.

$p = 1/Q$ = power factor of resonator (by coupling to waveguide).

$k = \sqrt{1-(r_2/r_1)^2}$ = eccentricity of focii in ellipse.

$F(k)$, $E(k)$ = complete elliptic integrals of first and second kinds.

$e = 2.72 = \exp 1$; $\ln e = 1$.

$j = \sqrt{-1}$ = quadrature operator.

sub-e = electric field.

sub-m = magnetic field.

sub-r = resonator.

sub-g = waveguide.

sub-c = effective (of body in field or hole in wall).

sub-c = cutoff (f or λ).

SC = short-circuit (electric wall).

OC = open-circuit (magnetic wall).

Note: Electric or magnetic polarizability is here defined with the dimensions of volume, as in an electrostatic or electromagnetic system of units; in general, it should include also the dimension of electrivity (electric permittivity) or magnetivity (magnetic permeability) of the medium.

III. Form of Presentation

In order to emphasize the significant properties of a coupling hole in a wall between two bounded regions, this presentation is limited to certain well defined situations, which are outlined in Table I. The theory is based on the limiting case of a coupling hole that is small as compared with the coupled regions, and as compared with the wavelength. Therefore the theory will be valid for small values of the ratios that will indicate the amount of coupling. Also, the coupling effects of the electric and magnetic fields can be separately evaluated. The wave medium and boundaries are assumed to have perfect properties, for example, walls of perfect conductivity.

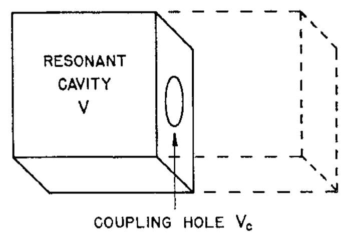
The ratios to be evaluated will be expressed as volume ratios or, in some cases, as area ratios. The latter will represent the volume ratios in cases of two-dimensional fields in three-dimensional space, or the area ratios may apply directly to the two-dimensional fields in a resistance sheet.

To this end, each field region and each coupling hole will be evaluated in terms of its effective volume or area with reference to the kind of field that is instrumental in effecting a certain amount of coupling.

TABLE I

Scope of This Presentation

Coupling between two regions bounded by conducting walls:
Two resonant cavities;
Two waveguides;
One resonant cavity and one waveguide.
Small hole, non-resonant:
Small coupling coefficient between two cavities;
Small normalized coupling reactance or susceptance between two waveguides;
Small loading power factor of cavity by coupling to waveguide.
Two or three-dimensional fields:
Coupling by magnetic field parallel to wall;
Coupling by electric field perpendicular to wall.



$$k = \frac{1}{4}\frac{V_c}{V}$$

Fig. 1—Coupling coefficient in terms of volume ratio.

Fig. 1 shows the principle of expressing the amount of coupling in terms of a ratio of two values of effective volume. Two resonant cavitities are separated by a common wall, and are coupled by a hole in this wall. For simplicity, the hole is so located that only one kind of field is effective therein, *e.g.*, the magnetic field. In a manner to be described, with reference to this kind of field, the effective volume of each cavity (V) and the coupling hole (V_c) will be defined and formulated. The coefficient of coupling between the cavities (k), as usually defined in circuit theory, is then expressed in terms of the ratio of these volumes, as indicated:

$$k = \frac{1}{4}\frac{V_c}{V}. \tag{1}$$

The factor $\frac{1}{4}$ is introduced by a concept based on a simple rule that will be stated.

The same principle will be applied to formulating the normalized reactance or susceptance (x or b) which may be effective in coupling two waveguides. Then, for the case of a resonant cavity coupled to a waveguide, a simple theorem will be presented for expressing the resulting "loading power factor" (p) of the resonator as a ratio of the values of effective volume.

While these principles have widest applicatiou to bounded regions of a wave medium, some of the ideas and relations can be presented most simply with reference to the two-dimensional field in a "resistance sheet." Particularly, such a sheet is most easily represented on paper. The field configuration therein is easy to conceive in terms of voltage gradient (volts/meter) and current density (amperes/meter) which are simply related by the resistance (ohms across a square). These are reasons why such a sheet is used in studies of analogous fields in three dimensions. Here it will be used in the introduction of some concepts and in the proof of some theorems.

It is recognized that the simplicity of the isotropic resistance sheet is available as a complete analogy, only for those three-dimensional cases which satisfy several conditions. In the region of interest, the medium must be isotropic and homogeneous, the field under consideration must be essentially a static field (not interacting with any other field), and the boundaries and field must be invariant in the third dimension. In this presentation, these conditions are all met with respect to the coupling field in the region of a small two-dimensional aperture. Furthermore, some of the principles based on the two-dimensional field are applicable to the three-dimensional field with corresponding restritions.

IV. Principles of Aperture Coupling in Two-Dimensional Fields

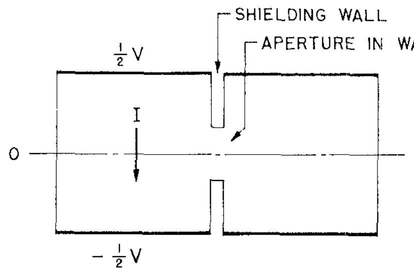
Several principles or rules are helpful in understanding the behavior of coupling holes and in expressing the amount of coupling. These will be presented here with reference to the two-dimensional fields in bounded areas or resistance sheet.

Fig. 2 illustrates the principle of duality. Each of two bounded regions is a rectangle of resistance sheet with conducting boundaries on two opposite edges and insulating boundaries on the other two edges. The two rectangles have a common edge, analogous to a shielding wall, and they are coupled by being joined over a small fraction of this edge, analogous to an aperture in the wall.

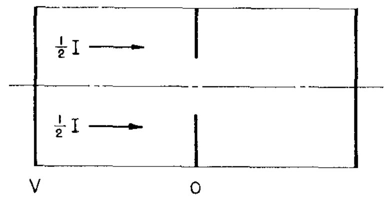
In Fig. 2(a), there is an insulating common wall, and the current (I) in the sheet is nominally parallel to the wall. The voltage (V) is applied between the two conducting walls. If one rectangle is excited as shown, there is some coupling through the aperture to the other rectangle. On the other hand, in Fig. 2(b), there is a conducting common wall, and the current in the sheet is nominally perpendicular to the wall.

These two cases are said to be related by "duality." This term denotes that one of the two orthogonal field patterns changes from potential contours to flux contours, and the other field pattern vice versa. Likewise, the conducting boundaries change to insulating boundaries, and vice versa. In the resistance sheet, voltage and current are interchanged.

If we take current to be analogous to flux, Fig. 2(a) shows the flux nominally parallel to the common wall, and Fig. 2(b) perpendicular. There are indicated, for each relationship, the analogous cases of electric-field coupling and magnetic-field coupling. Of the resulting four cases, only two are of particular interest for a wave medium, the two requiring a conducting wall, because such a wall is the kind that can be realized as a shield



(a) Flux (I) parallel to wall:
Electric coupling through aperture in insulating wall (shown); Magnetic coupling through aperture in conducting wall (analog).



(b) Flux (I) Perpendicular to wall:
Electric coupling through aperture in conducting wall (shown);
Magnetic coupling through aperture in insulating wall (analog).

Fig. 2—Aperture coupling between two rectangles of resistance sheet; duality.

for both electric and magnetic fields. (A conducting wall is a shield against an high-frequency magnetic field, if the "skin-depth" is a small fraction of this thickness.) On the other hand, the hypothetical insulating wall is not easily realizable for a wave medium, because the wall would be required to have much smaller electrivity (permittivity) and much greater magnetivity (permeability), both of which are not generally found in available materials.

In Fig. 2, the shapes shown have symmetry about the indicated centerlines. In Fig. 2(a), the centerline is at zero potential, so it may be the location of a conducting boundary for enabling one half of the sheet to have the same field configuration. In Fig. 2(b), the centerline is crossed by no current, so it may be the location of an insulating boundary.

Fig. 3 shows qualitatively the orthogonal field contours in a pair of equal rectangles with aperture coupling. One set of contours is shown above the centerline and the other set below the centerline. The exciting voltage or current is applied across the left-hand rectangle, which therefore has its flux nominally parallel or perpendicular to the common wall. The right-hand rectangle is excited by the coupling through the aperture in this wall.

For the shape illustrated in Fig. 3, the coupling coefficient (k) happens to be about $\pi/64$ or 0.049; the corresponding contour is marked for emphasis. Either the voltage or the current is coupled in this ratio, since the two rectangles are equal.

There is a simple "rule of one half" that will play an essential role in this presentation. This rule, which has

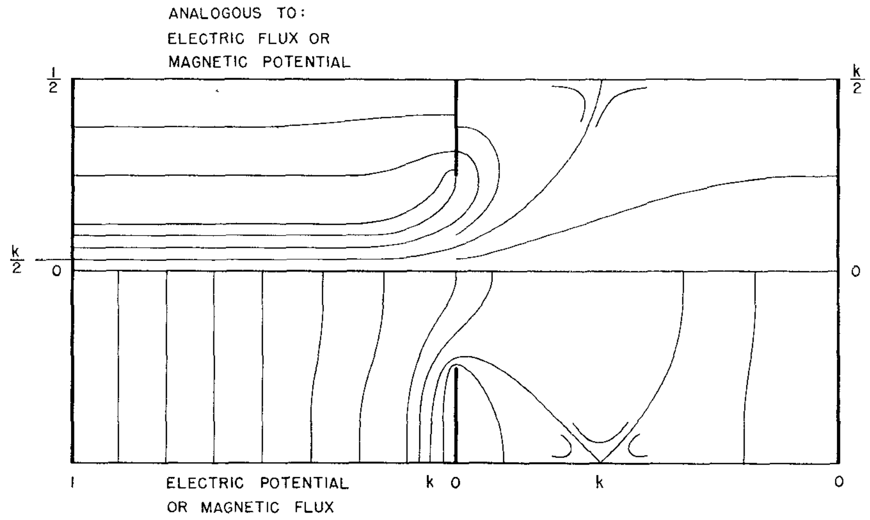
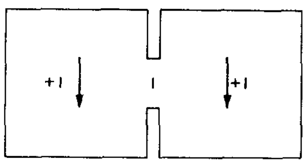
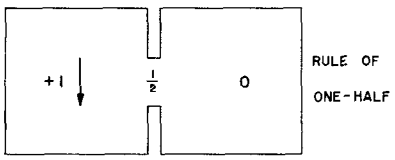


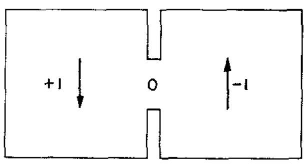
Fig. 3—Field contours for aperture coupling between two rectangles.



(a) Symmetric.



(b) One side, average of (a) and (c).



(c) A symmetric.

Fig 4—Rule of one-half field intensity in center of a small aperture in a thin wall.

been recognized in various physical situations, will be stated and proved with reference to Fig. 4. It applies strictly to a symmetrical small aperture in a plane thin wall of perfect shielding. For present purposes, the aperture is symmetrical with respect to two mutually perpendicular axes, at least one of which is perpendicular to the field direction. The aperture is much smaller than its distance to the nearest boundary other than the wall in which it is located. The rule is valid alike for two- or three-dimensional fields.

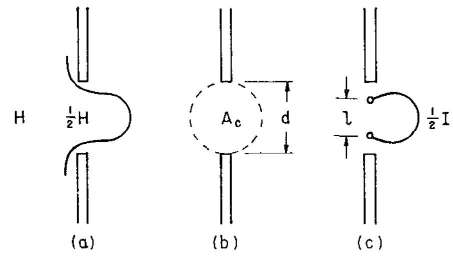
Fig. 4 shows two regions coupled through an aperture in a common shielding wall, in accordance with the stated conditions. The wall is taken to have such properties that it is invisible to a parallel field, as exemplified by an insulating slit in a resistance sheet. In the left-hand region, the field intensity is $+1$, except for the vicinity of the aperture. In the right-hand region, the field intensity is respectively $+1$, 0, -1, the intermediate case being the average of the two extreme cases. In one extreme case, Fig. 4(a), the common wall is invisible, so the field intensity in the aperture is $+1$. In the other extreme case, Fig. 4(c), the fields cancel out in the center of the aperture. The intermediate case, Fig. 4(b), is simply the average of the other two cases, so the field intensity is $\frac{1}{2}$ in the center of the aperture. Therefore the field intensity in the region on either side of the aperture is represented by $\frac{1}{2}$ of this field intensity in the center of the aperture. This is here designated as the "rule of one half."

In Fig. 4, the field direction is parallel to the shielding wall, and the wall is such as to be invisible to a uniform field of this kind and direction. This is true of a resistance sheet with an insulating wall and with current in the indicated directions. However, an analogous proof is applicable to any field component that is parallel or perpendicular to a shielding wall of any kind. For the usual conducting wall, the cases of most interest are the parallel magnetic field or the perpendicular electric field, as outlined in Fig. 2.

The classic derivation by Bethe (see [4], [5]) is expressed in terms of the "polarizability" of an aperture, for evaluating the coupling from one side to the other. Taking the resistance sheet as a region of two-dimensional field, this concept will be presented with reference to Fig. 5.

The resistance sheet is divided by an insulating slit representing a thin shielding wall in which there is an aperture of small width (d). Referring to Fig. 5(a), the exciting current density on the left-hand side is parallel to the wall and has a certain value (H). This value is reduced by the "rule of one half" in the center of the aperture ($\frac{1}{2}H$). Some of the field lines loop through the aperture into the right-hand side. On that side, far from the aperture, the field is the same as that of a small current dipole located at the center of the aperture, as indicated in Fig. 5(c). This equivalent dipole has a certain moment (Il).

In accordance with previous uses of the term, Bethe



$$\text{dipole moment} = Il = H\left(\frac{\pi}{16} d^2\right)$$

$$\text{polarizability} = \frac{Il}{H} = \frac{\pi}{16} d^2 = \frac{1}{4} A_c$$

$$\text{effective area} = A_c = \frac{\pi}{4} d^2 = \text{circle}$$

Fig. 5—Equivalent dipole moment of coupling through an aperture, as a basis for defining its polarizability.

implicitly defined the "polarizability" of an aperture as the quotient of the equivalent dipole moment over the incident field intensity. In his use of electrostatic and electromagnetic units, this quotient appeared to have simply the dimensions of volume. In an integrated electromagnetic system of units (exemplified by the mks system), this quotient would include also the relevant property of the medium (electrivity or magnetivity). Here, in order to retain the dimensional simplicity, we redefine polarizability as the quotient of the equivalent dipole moment over the flux density of the field. Then it has the dimensions of area or volume in a two- or three-dimensional field. Here the polarizability will be related to the "effective" area or volume of an aperture.

In the resistance sheet, as shown in Fig. 5, the polarizability is formulated as the quotient of the equivalent dipole moment (Il) over the current density (H), to give the dimensions of area. For a conducting wall in a wave medium, this is closely analogous to the coupling through a long slot of constant width (d) excited by a transverse magnetic field. By duality, it is analogous to the coupling through the same slot excited by an electric field perpendicular to the wall. These cases both give the same area of polarizability, with respect to a two-dimensional field.

Referring further to Fig. 5, the equivalent dipole moment and the polarizability are formulated from Bethe's derivation for the two-dimensional magnetic field across a long slot in a conducting wall. Here we note that the polarizability is $\frac{1}{4}$ the area of a circle inscribed in the aperture (A_c). By the "rule of one half," the center of the aperture is subject to $\frac{1}{2}$ the field intensity on either side, so we may say that the "effective area" for interaction would be subject to $(\frac{1}{2})^2$ or $\frac{1}{4}$ the product of the field intensities. On this basis, the polarizability becomes equal to $\frac{1}{4}$ the effective area. In this case, the effective area (A_c) is simply the inscribed circle, as indicated in Fig. 5(b). This concept is in accord with one's intuitive notions of the interaction in the aperture region.

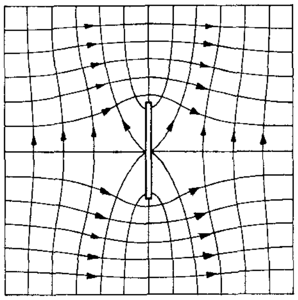
The concept of polarizability was originated to evaluate the effect of a body introduced into a uniform field, such as a molecule in a dielectric or a loading conductor in an artificial dielectric. Some cases of the latter are directly related to coupling apertures by duality and Booker's principle (see [6], [10], [14]). Specifically, a thin body in a uniform field is analogous to an aperture in a thin wall, under certain conditions.

Fig. 6 shows qualitatively the two-dimensional field contours of the same region with opposite boundaries on the vertical center line. These may be regarded as a resistance sheet or a two-dimensional cross section of a three-dimensional region. The terminology will be descriptive of the latter.

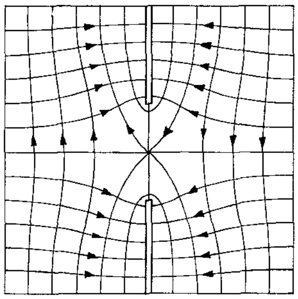
Fig. 6(a) shows a thin conducting strip in a uniform field, either an electric field parallel to the plane of the strip or a magnetic field perpendicular, these being related by duality. Fig. 6(b) shows a slot of the same width in a thin conducting wall, with either a magnetic field parallel to the wall or an electric field perpendicular, these also being related by duality. In Fig. 6(a), the field is so oriented that there is zero field in the center of the strip. In Fig. 6(b), this result is obtained by opposite field directions on the opposite sides of the wall. Then the field configurations acquire the same shape. This relationship between Figs. 6(a) and 6(b) is a further application of duality, but especially as presented by Booker in his classical analogy between a conducting strip and a slot in a conducting sheet (see [6]).

By successive steps, there will be established a quantitative relation between a thin body in a uniform field and an aperture in a thin wall. In particular, they will be shown to have the same effective area or volume, in the terms of this presentation.

Fig. 7 shows a basis for evaluating the effective area (or volume) of a thin body in a uniform field. The case shown is based on Fig. 6(a), the cross section of a thin



(a) Insulating wall, conducting strip: Parallel electric field or Perpendicular magnetic field.



(b) Conducting wall, insulating slot: Parallel magnetic field or Perpendicular electric field.

Fig. 6—Field contours of a thin body in a uniform field, and the analogous coupling in a thin wall; duality and Booker's principle.

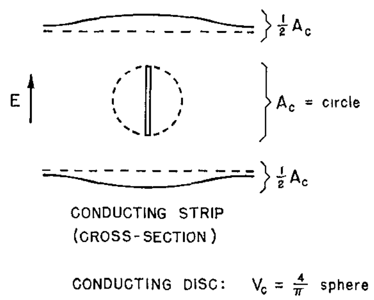


Fig. 7—Evaluation of effective area (or volume) of a thin body in a uniform field.

conducting strip in an electric field. At any distance from the strip, the perturbation from the uniform field appears as a spreading of the potential contours. Taking any pair of contours at the same distance from the strip, the amount of spreading may be measured by the increment of the cross-sectional area included between these contours. As this area is evaluated further from the strip, it approaches a limiting value (A_c) which is defined as the "effective area." In this case, it is found to be equal to the circle circumscribed on the width of the strip, as indicated (see [17], [21]).

If instead we take a thin circular disk in Fig. 7, its "effective volume" (V_c) on the same basis is $4/\pi$ times the circumscribed sphere (see [4], [5], [17], [21]).

From Figs. 5 and 7 for two-dimensional fields, we are prepared to compare an aperture in a thin wall with the analogous thin body in a uniform field. We note that they have the same effective area, as here defined. A similar conclusion may be reached from a consideration of the known formulas for three-dimensional fields, giving the same effective volume for the aperture in a thin wall and the analogous thin body in a uniform field. In any case, this like evaluation is enabled by the concept of applying the "rule of one half" to the fields on both sides of the aperture, and thereby introducing the factor $\frac{1}{4}$ in (1).

Referring to Fig. 7, the significance of the effective volume (V_c) may be exemplified by the effect of a small thin conducting disk introduced into the region of uniform electric field in an idealized parallel-plate capacitor. If this region has a certain volume (V), the relative increment of capacitance is

$$\frac{\Delta C}{C} = \frac{V_c}{V}. \tag{2}$$

In this case, the polarizability, as defined herein, is equal to the effective volume.

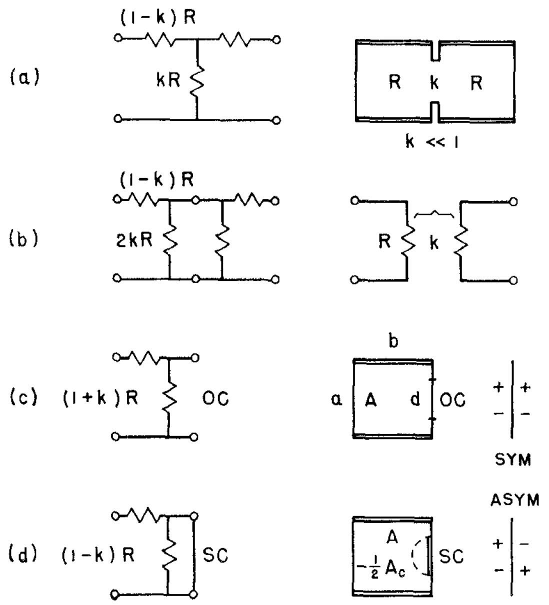
While the relations stated above are all based on derivations found in the literature, there is one proof that will be given as a link between the effective volume of a thin body and that of an aperture in a thin wall. This proof will be based on circuit theory with the aid of the bisection theorem (see [1], [2]).

Fig. 8 illustrates this proof relating to aperture coupling. It is intended to shed further light on the concept of effective area (or volume). It is based on a symmetric pair of rectangles of resistance sheet and the equivalent resistance network, as shown in Fig. 8(a). The rectangles have in common an insulating wall, part of which is removed to leave the two rectangles in contact over a width of aperture. The upper and lower conducting walls of each rectangle form one pair of terminals of the network. Each rectangle presents at its pair of terminals a resistance (R) if the other pair is left unconnected. The aperture provides an apparent coupling resistance (kR) as indicated in the equivalent network. The aperture is assumed so small that the coupling coefficient is much less than unity ($k \ll 1$).

Fig. 8(b) shows two other network representations, one of these being bisected for further analysis. The objective is to formulate the coupling coefficient in terms of the dimensions of the rectangles and their coupling aperture.

Fig. 8(c) shows one half of the network with open circuit (OC) at the bisection terminals. This corresponds to one rectangle with the insulating wall completed so there is no current diverted into the other rectangle. Fig. 8(d) shows the same except with short circuit (SC) at the bisection terminals. This corresponds to one rectangle with a conducting wall substituted for the aperture.

Since the aperture does not have terminals in the literal sense, a rigorous equivalence in Figs. 8(c) and (d) requires the network concept of symmetric and asymmetric "modes," as indicated beside the rectangle apertures. These concepts are based on simultaneous excita-



$$\frac{1-k}{1+k} = \frac{A - \frac{1}{2}A_c}{A}; \qquad 1 - 2k = 1 - \frac{A_c}{2A}$$

$$k = \frac{A_c}{4A} = \frac{\pi d^2}{16ab}$$

Example (as drawn): $a = b = 2d; \quad k = \dfrac{\pi}{64} = 0.049$

efficiency $= \frac{1}{4}k^2$ (approx.) $= 0.00060 = -32.2$ db

Fig. 8—Equivalent-network derivation of the coupling coefficient between two rectangles, in terms of the effective area of the coupling aperture.

tion of both sides of the symmetric network, in the same or opposite polarities. Such a consideration establishes the equivalence of an open circuit or a short circuit and, respectively, an insulating or a conducting wall at the coupling aperture.

Referring to the change from Fig. 8(c) to (d), we see that the effect of the aperture appears in the form of a conducting body introduced into an otherwise uniform field, as discussed with reference to Figs. 5–7. In Fig. 8(d), the rectangle area ($A = ab$) includes only half the effective area of the conducting body ($\frac{1}{2}A_c = (\pi/8)d^2$). Comparing the OC and SC conditions, the relative change of resistance in the network ($2k$) is equal to that in the rectangle ($\frac{1}{2}A_c/A$) as indicated. This equality establishes the $\frac{1}{4}$ factor previously introduced.

This example behaves as a resistive attenuator. Its behavior may be expressed in terms of the maximum efficiency of coupling between generator and load, which is

$$\text{efficiency} = \left(\frac{k}{1+\sqrt{1-k^2}}\right)^2 = \frac{1}{4}k^2 \ll 1. \tag{3}$$

For the shape shown, the computed coupling and efficiency are indicated, corresponding to an attenuation of 32.2 db.

V. Various Shapes of Coupling Holes

The preceding discussion has been directed mainly to examples of two-dimensional fields, in which the aperture has only one dimension (width) and no option as to shape. In three-dimensional fields, there are several shapes of coupling hole that are interesting in theory or practice.

Fig. 9 shows several shapes of coupling holes, with their relative values of effective volume (V_{mc}) for magnetic field parallel to the major axis. The relative values are referred to unity for the simplest case of a circle, and are based on equal values of the major diameter ($d = 2r_1$). The shapes are self-explanatory, being made of straight lines and circular arcs except for the ellipse. Only the ellipse (of which the circle is a special case) can easily be computed. All have been described and evaluated theoretically or experimentally by Cohn (see [14]–[16], [21]). The general properties of these shapes are familiar and need no comment.

The ellipse offers a family of cases that exemplify a range of shapes, of which some extreme cases are especially interesting and significant. They are unusual in their simplicity of mathematical formulation. Table II gives the effective volume (V_c) as defined here for an elliptic disc or hole with respect to any of the three field directions of interest. In addition to the general formula in terms of complete elliptic integrals, the extreme cases of a circle and a narrow ellipse are formulated in terms of more elementary functions. In each case, the coefficient in parentheses will be recognized as the volume of a sphere or spheroid circumscribed on the ellipse.

Referring to Table II, there are several features worthy of note. For the narrow ellipse, much the greater effective volume is presented to the longitudinal magnetic field. For the transverse magnetic and the perpendicular electric fields, the narrow ellipse approximates a long strip or slot with a two-dimensional field, so these two cases are related by duality and hence present the same effective volume. For the circle, there is no distinction between the two magnetic field orientations. Here we note the ratio of 2:1 between magnetic and electric polarizabilities of a circular hole which was discovered and put to use by Bethe (see [4], [5]).

Most of the present discussion applies quantitatively to the limiting case of a hole in a thin wall. In practice, the wall usually has a thickness sufficient to reduce the coupling by an appreciable amount. In principle, this reduction is related to the field attenuation in a cylindrical shield that is too small for wave propagation (see [8], [9]). Fig. 10 gives some relations that are helpful in estimating this effect. Taking a cylinder having a cross section like the hole in size and shape, the exponential rate of attenuation along the cylinder can be evaluated. In the short cylinder through the thick wall, this rate gives an upper limit on the attenuation of the coupling by the thickness of the wall. The rate is indicated for several simple cases of two- and three-dimensional fields.

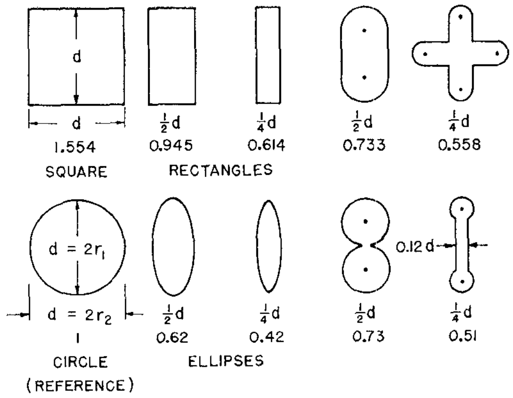
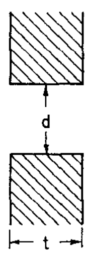


Fig. 9—Various shapes of coupling holes and relative values of effective volume for magnetic field parallel to major axis.

TABLE II

FORMULAS FOR THE EFFECTIVE VOLUME OF AN ELLIPTIC DISC OR HOLE

Field			Circle	General	Narrow
Direction	Disc	Hole	$r_2 = r_1$	$k = \sqrt{1 - (r_2/r_1)^2}$	$r_2 \ll r_1$
Major Axis $2r_1$	E	H	$\underbrace{\left(\frac{4\pi r_1^3}{3}\right)}_{\text{Sphere}}\frac{4}{\pi}$	$\left(\frac{4\pi r_1^3}{3}\right)\frac{k^2}{F(k) - E(k)}$	$\left(\frac{4\pi r_1^3}{3}\right)\frac{1}{\ln\frac{4r_1}{er_2}}$
Minor Axis $2r_2$	E	H		$\left(\frac{4\pi r_1 r_2^2}{3}\right)\frac{k^2}{E(k) - (1 - k^2)F(k)}$	$\left(\frac{4\pi r_1 r_2^2}{3}\right)$
Perpendicular	H	E	$\overbrace{\left(\frac{4\pi r_1^3}{3}\right)}\frac{2}{\pi}$	$\left(\frac{4\pi r_1 r_2^2}{3}\right)\frac{1}{E(k)}$	



Two-dimensional field: Parallel magnetic field; Perpendicular electric field.

$$A_c = \frac{\pi}{4} d^2 \exp - \pi \frac{t}{d}$$

Three-dimensional field, circular hole: Parallel magnetic field.

$$V_{mc} = \frac{2}{3} d^3 \exp - 3.68 \frac{t}{d}$$

Perpendicular electric field.

$$V_{ec} = \frac{1}{3} d^3 \exp - 4.81 \frac{t}{d}$$

Fig. 10—Aperture in thick wall.

VI. Coupling Between Resonant Cavities

Having developed the concept and significance of the effective area or volume of a coupling hole, its simplest and most interesting application is found in the coupling between two resonant cavities, as introduced in Fig. 1. Here the two cavities are taken to be alike, and bounded by walls of perfect conductivity with resulting perfect shielding. Quantitative evaluation will be based on the assumption of a small coupling hole in a thin common wall.

The simplest resonant cavity for this purpose is the "square pillbox" shown in Fig. 11 (next page). This designation implies that the third dimension (c = height, not shown) may be much smaller than the diameter (a). In any case, the field is taken to be two dimensional as in the rectangular TM-110 mode of resonance. Incidentally, this requires that the coupling hole have the same width (d) over the full height of the cavity. The two cavities with a common side wall for the coupling hole are coupled by magnetic field in the hole.

As one step, it is necessary to evaluate the effective area or volume of the resonant cavity. In general terms, the electric field is greatest in the center of the square and the magnetic field is greatest in the middle of each side wall. The hole is located in the middle of one side wall, so the magnetic field intensity (H) in this location (but before the hole is opened) is taken as a reference. The effective area of the resonant cavity is defined as the area (A_m) which would contain the same amount of magnetic energy if filled with uniform intensity equal to the reference value (H). In the square cavity, this is $\frac{1}{2}$ the area, as indicated. It is interesting to designate, as shown in dotted lines, the parts of the area that contain most of the magnetic energy. If we apply the same principle to the electric field, taking the maximum value (E) as a reference, its effective area (A_e) is $\frac{1}{4}$ of the square, this energy being contained mostly in the center area, shown in dotted lines. The areas in the corners contain little energy of either kind.

In Fig. 11, the hole for magnetic coupling has an effective area (A_{mc}) equal to the inscribed circle, as previously shown in Figs. 5 and 7. The coupling coefficient (k_m) is then simply formulated as $\frac{1}{4}$ the ratio of the effective area of the hole over that of the cavity. This follows the reasoning of Fig. 8 and the preceding discussions.

In all evaluations in this paper, the assumption of a small hole limits the validity to a coupling coefficient much less than unity. In Fig. 11, for example, a close approximation would require a hole less than $\frac{1}{4}$ the diameter ($d/a<\frac{1}{4}$, $k_m<\pi/128=0.025$); a rough approximation would be obtained for a hole less than $\frac{1}{2}$ the diameter ($d/a<\frac{1}{2}$, $k_m<\pi/32=0.10$). The residual relative error is of the order of the coupling coefficient (k_m).

Fig. 12 shows the magnetic coupling on the same basis for two resonant cavities of the "rectangular-pillbox" type. Here the formulas are complicated by the reference intensity (H) at two walls (b) being different from the intensity at the other two walls (a).

Fig. 13 shows the magnetic coupling for two resonant cavities of the "circular-pillbox" type. Here there is no "common" wall in the literal sense, but this is approximated in the vicinity of a small hole. The circular TM-010 mode of resonance is utilized. Here it is remarkable that the effective area of magnetic energy is equal to the area of the cavity; the small density near the center is compensated by the excess of density in an intermediate region just inside the rim. The effective area of the hole likewise being a circle, the result is an extremely simple formula for the coupling coefficient.

Fig. 14 shows the magnetic coupling for two resonant cavitities of the "coaxial-circular-pillbox" type. For evaluation, this is assumed much smaller than the wavelength, so the magnetic and electric fields are substantially separated in the outer and inner regions, designated as inductance (L) and capacitance (C).

In the related case of a pair of adjacent long coaxial lines coupled by a longitudinal slot of length much greater than its width, we can likewise express the coupling coefficient that is effective over the length of the slot. By duality, it has the same value for both magnetic and electric fields ($k_m=k_e$), and is given by the formula in Fig. 14. In this case, there is no assumption of resonance, nor is it ruled out.

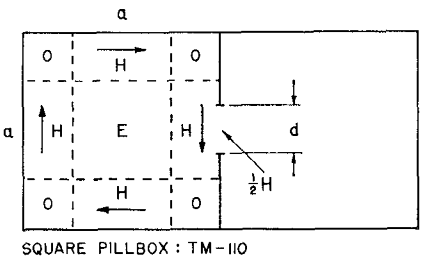
Going to three-dimensional fields, the simplest case is the coupling of two cubic resonant cavities by a circular hole, as shown in Fig. 15. The resonance is in the pillbox mode, rectangular TM-110, which has a two-dimensional field. However, the coupling hole is circular, which departs from the two-dimensional field.

Fig. 15(a) is an extension of Fig. 11, the magnetic-coupling hole being changed to a circle. Using effective volume instead of area for both the cavity and the hole (V_m, V_{mc}), the coupling coefficient (k_m) is simply formulated.

Fig. 15(b) has the cavity field reoriented for electric coupling, the hole being located at the maximum electric field intensity E. From Fig. 11 and Table II, it is found that the effective volume of the cavity and that of the hole are both reduced to one half, so the coupling coefficient remains the same. This is a remarkable coincidence, which might be useful in the design of such cavities for simultaneous utilization of different orthogonal modes at the same frequency (or nearby frequencies).

Fig. 16 is changed from Fig. 15(a) in the same manner as Fig. 12 from Fig. 11; two rectangular resonant cavities are coupled by a circular hole.

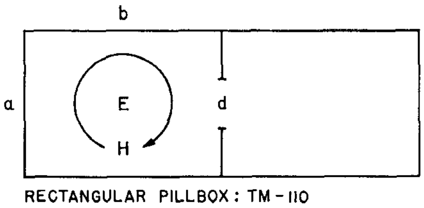
These examples of coupling holes between various types of resonant cavitities have been chosen to illustrate some principles and to give some formulas for simple cases that may be approximated in practice. They can be extended to other cases in terms of the values of effective area or volume for any particular cavities and coupling hole.



$$A_m = \frac{1}{2}a^2; \quad A_{mc} = \frac{\pi}{4}d^2; \quad k_m = \frac{A_{mc}}{4A_m} = \frac{\pi}{8}\left(\frac{d}{a}\right)$$

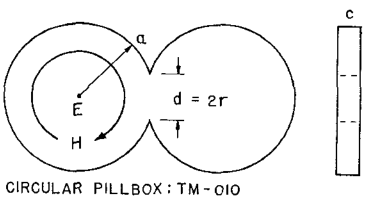
$$A_e = \frac{1}{4}a^2$$

Fig. 11—The effective area of the stored energy in a square-pillbox resonant cavity; the coefficient of coupling between two such cavities.



$$A_m = \frac{ab}{2}\,\frac{a^2+b^2}{2a^2}; \quad k_m = \frac{\pi}{8}\,\frac{d^2}{ab}\,\frac{2a^2}{a^2+b^2}$$

Fig. 12—Rectangular-pillbox resonant cavities.

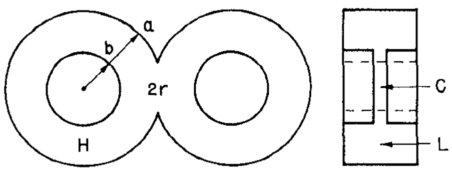


$$A_m = \pi a^2 = \text{circle } (a)$$

$$A_{mc} = \pi r^2 = \text{circle } (r)$$

$$k_m = \frac{A_{mc}}{4A_m} = \frac{1}{4}\left(\frac{r}{a}\right)^2$$

Fig. 13—Circular-pillbox resonant cavities.

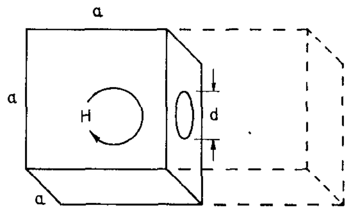


$$A_m = 2\pi a^2 \ln\frac{a}{b}; \quad k_m = \frac{1}{8\ln\dfrac{a}{b}}\left(\frac{r}{a}\right)^2$$

Coaxial line, over length of narrow slot:

$$k_e = k_m \text{ (above)}$$

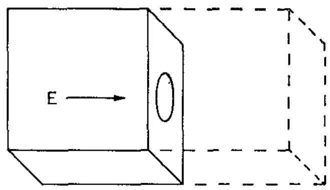
Fig. 14—Coaxial-circular-pillbox resonant cavities.



(a) Magnetic coupling.

$$V_m = \frac{1}{2}a^3; \quad V_{mc} = \frac{2}{3}d^3 = \frac{4}{\pi} \text{ sphere}$$

$$k_m = \frac{V_{mc}}{4V_m} = \frac{1}{3}\left(\frac{d}{a}\right)^3$$

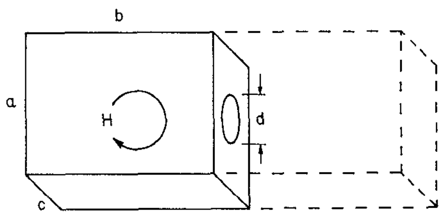


(b) Electric Coupling.

$$V_e = \frac{1}{4}a^3; \quad V_{ec} = \frac{1}{3}d^3 = \frac{2}{\pi} \text{ sphere}$$

$$k_e = \frac{V_{ec}}{4V_e} = \frac{1}{3}\left(\frac{d}{a}\right)^3 = k_m$$

Fig. 15—Cubic resonant cavities coupled by a circular hole.



$$V_m = \frac{abc}{2}\,\frac{a^2+b^2}{2a^2}; \quad V_{mc} = \frac{4}{\pi}\left(\frac{\pi}{6}d^3\right) = \frac{2}{3}d^3$$

$$k_m = \frac{V_{mc}}{4V_m} = \frac{4}{3}\,\frac{d^3}{abc}\,\frac{2a^3}{a^2+b^2}$$

Fig. 16—Rectangular resonant cavities coupled by a circular hole.

VII. Coupling Between Waveguides

Two waveguides may be coupled in various ways through a hole in a common wall. In order to apply the principles presented here, we must evaluate the effective volume of the waveguide for comparison with that of the hole. Unlike the resonant cavity, the waveguide does not have a limited region in which to evaluate the effective volume for this purpose. Therefore, we shall arbitrarily define an effective volume such that it will fit into a certain formula.

A simple case of coupling between two rectangular waveguides is shown in Fig. 17 (next page). It has a circular hole in the common end wall of the two waveguides. Alternatively, it may be regarded as an iris inserted in a continuous waveguide. The amount of intercoupling may be expressed in terms of the "normalized reactance" ($x=X/R_0$) in shunt across the waveguide. Reactance is chosen rather than susceptance, because it is zero for a complete shield with no hole and increases with coupling between the two waveguides. The normal mode in this waveguide is the rectangular TE-10 mode; on reflection at the end wall, it develops a maximum magnetic field (H) at the end wall midway between the side walls (b). This value is used as the reference, because the hole is to be opened at this location.

Corresponding to (1) for the coupling coefficient between resonant cavities, it is desired to express the normalized reactance in the same form,

$$x = \frac{1}{4}\frac{V_c}{V}. \tag{4}$$

Here the effective volume of the waveguide (V) remains to be defined.

In Fig. 17, the normalized reactance is known in terms of dimensions (see [7], [12]), as is the effective volume of the circular hole for magnetic coupling (V_{mc}), so the effective volume of the waveguide (V_m) can be formulated as shown. It is interesting, and presumably significant, to associate this computed volume with some region in the waveguide; this is done in dotted lines on the diagram. The length of this region $(1/2\pi)\lambda_g$ is recognized as one radianlength in the waveguide.

There are some purposes to be served by evaluating the effective volume in the waveguide. It can be expressed and remembered in simple dimensions independent of the coupling hole, and subsequently can be used in a ratio with any shape of coupling hole. It varies with guide wavelength λ_g and hence with frequency, causing a corresponding inverse variation in the normalized reactance, as indicated in the formulas in Fig. 17. It may be helpful in appreciating the significance of the fields in space as determining the amount of coupling through the hole.

Fig. 18 shows the circuit representation of the two ways in which two waveguides may be coupled by normalized reactance (x). Fig. 18(a) shows the end-wall coupling between two colinear waveguides, as just described, while Fig. 18(b) shows the top-wall coupling between two parallel waveguides. In the latter form, the common wall is the top wall of one rectangular guide and the bottom wall of the other. In both forms, the normalized reactance and the effective volume have the same values. In Fig. 18(b), the mutual reactance is negative because the representation is made in terms of coupling between the upper conductors of both circuits, whereas the waveguide coupling is between the upper conductor of one guide and the lower conductor of the other. The self-reactance shown in each circuit represents the increment caused by opening the hole in the wall.

Fig. 19 shows the circuit representation of the two ways in which two parallel waveguides may be coupled by normalized susceptance (b). Fig. 19(a) shows side-wall magnetic coupling while Fig. 19(b) shows top-wall electric coupling. By the usual methods, the normalized susceptance (b_m, b_e) is evaluated and this enables formulation of the effective volume (V_m, V_e) for these configurations as was done in Fig. 17.

In Fig. 19, the circuit representations indicate the magnitude of the normalized susceptance by the symbol (b). The positive prefix (j) indicates positive capacitance or negative inductance, whereas the negative prefix ($-j$) indicates negative capacitance or positive inductance. The self-susceptance shown in each circuit represents the increment caused by opening the hole. It is negative inductance (jb) in Fig. 19(a) or negative capacitance ($-jb$) in Fig. 19(b). In Fig. 19(a), the coupling susceptance is positive inductance ($-jb$). In Fig. 19(b), the coupling susceptance is negative capacitance ($-jb$) because the representation is made in terms of coupling between the upper conductors of both circuits, whereas the waveguide coupling is between the upper conductor of one guide and the lower conductor of the other.

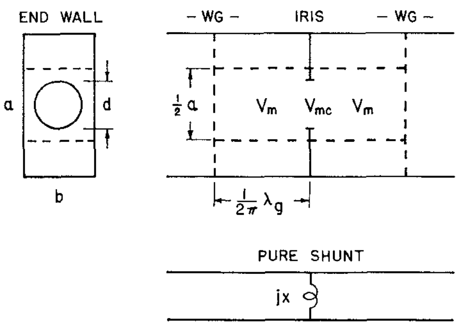
Figs. 18(b) and 19(b) show magnetic and electric top-wall coupling that would occur simultaneously in different amounts. The ratio of these couplings would then be significant, as determined by the relative effective volume of the hole and waveguides with respect to these fields.

$$\frac{x_m}{b_e} = \frac{V_{mc}}{V_{ec}}\frac{V_e}{V_m} = 2\left(\frac{\lambda_0}{\lambda_g}\right)^2 = 2\sqrt{1-(f_c/f)^2}. \tag{5}$$

In the usual operating range of the waveguides, this ratio exceeds unity, so the magnetic coupling is dominant. It is noted that these two couplings give a directional coupling from a wave in one guide to waves in both directions in the other guide, in the manner that is well known.

In further reference to Figs. 18(b) and 19(b), top-wall coupling is generally mixed magnetic and electric coupling, unless one kind is intentionally selected against the other. For example, a narrow ellipse favors coupling by longitudinal magnetic field, as seen in Table II, so a narrow slot is commonly used to subordinate electric coupling.

Among the various couplings in Figs. 18 and 19, only one has zero slope in any part of the usual operating frequency range. Fig. 19(b) has minimum susceptance at a certain frequency ($f/f_c=\sqrt{2}$); the reason is the tendency of this coupling to increase toward higher frequency and also toward the cutoff frequency. This electric coupling alone cannot be obtained by a simple coupling hole, but it could be obtained by a probe through a small hole designed to subordinate magnetic coupling.



$$x = \frac{V_{mc}}{4V_m} = \frac{\pi}{12}\left(\frac{d}{b}\right)^3\left(\frac{2b}{a}\right)^2\left(\frac{2a}{\lambda_g}\right)$$

$$= \frac{\pi}{12}\left(\frac{d}{b}\right)^3\left(\frac{2b}{a}\right)^2\sqrt{(f/f_c)^2 - 1}$$

$$V_{mc} = \frac{2}{3}\,d^3; \qquad V_m = \left(\frac{1}{2}\,a\right)(b)\left(\frac{1}{2\pi}\,\lambda_g\right) = \frac{ab\lambda_g}{4\pi}$$

Fig. 17—Normalized reactance between waveguides; coupling hole in end wall.

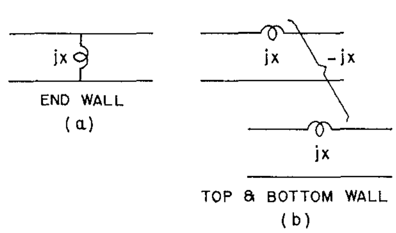
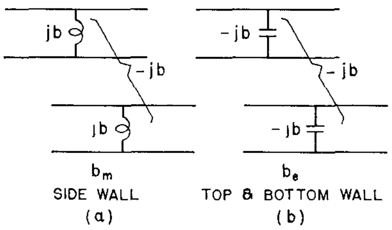


Fig. 18—Normalized reactance between waveguides; coupling hole in end wall or top wall.



$$V_m = \left(\frac{1}{2}\,a\right)(b)\left(\frac{1}{2\pi}\,\lambda_g\right)\left(\frac{2a}{\lambda_g}\right)^2 = \frac{ab\lambda_c^2}{4\pi\lambda_g}$$

$$V_e = \left(\frac{1}{2}\,a\right)(b)\left(\frac{1}{2\pi}\,\lambda_g\right)\left(\frac{\lambda_0}{\lambda_g}\right)^2 = \frac{ab\lambda_0^2}{4\pi\lambda_g}$$

$$b_m = \frac{V_{mc}}{4V_m} \propto \lambda_g \propto \frac{1}{\sqrt{(f/f_c)^2 - 1}}$$

$$b_e = \frac{V_{ec}}{4V_e} \propto \frac{\lambda_g}{\lambda_0^2} \propto \frac{f^2}{\sqrt{(f/f_c)^2 - 1}}$$

$$b_e = \text{minimum at } f/f_c = \sqrt{2}$$

Fig. 19—Normalized susceptance between waveguides; coupling hole in side wall or top wall.

VIII. Coupling Between Resonant Cavity and Waveguide

The evaluation of the coupling between two like cavities or waveguides is simplified by symmetry. This symmetry is lost in the coupling between a resonant cavity and a waveguide. A theorem has been discovered which enables a simple evaluation in this unsymmetric case from a knowledge of the two symmetrical cases. This theorem will be presented and proved with reference to Figs. 20 and 21.

The coupling between a resonant cavity and a waveguide is usually expressed by the "loading power factor" ($p = 1/Q$) of the resonator loading by its coupling with one end of a waveguide. The latter is assumed to have a nonreflecting termination at its other end.

Fig. 20 shows the circuit representation of a resonator coupled with a nonreflecting waveguide by a relatively small value of coupling reactance. The latter may be expressed in either of two ways. From the viewpoint of the coefficient of coupling between two like resonators, it is expressed in one form (kX). From the viewpoint of the normalized reactance between two like waveguides, it is expressed in another form (xR_0). The resistance (R) which is coupled into the resonator is then expressed in two ways, and the ratio mean of the two expressions comes out in simple form. This yields an extremely simple expression for the loading power factor:

$$p = kx. \tag{6}$$

The significance of this relation will be described, and the loading power factor will be evaluated in terms of volume ratios.

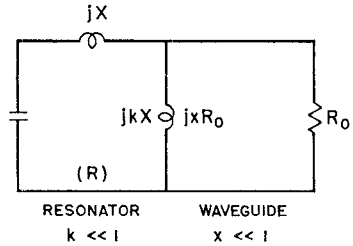
Fig. 21(a) shows a resonator of a certain effective volume (V_r) coupled with a waveguide of a certain effective volume (V_g) through a hole in the end wall, having a certain effective volume (V_c). Each effective volume is evaluated with reference to the same kind and orientation of field in the vicinity of the hole, in the manner here presented.

The resonator and hole of Fig. 21(a) can be imaged to form two like resonators coupled by the same hole, as shown in Fig. 21(b). This symmetric arrangement is used for defining and evaluating a coupling coefficient (k).

The waveguide and hole of Fig. 21(a) can be imaged to form two like waveguides coupled by the same hole, as shown in Fig. 21(c). This symmetric arrangement is used for defining and evaluating a normalized reactance (x).

With the coupling coefficient and the normalized reactance so defined, their product becomes equal to the loading power factor in Fig. 21(a). It may be formulated in terms of the corresponding volume ratios, as indicated.

Fig. 22 shows an example of the loading power factor by coupling between a resonant cavity and a waveguide. The dimensions are chosen for a convenient relation be-



In resonator:

$$R = \frac{(xR_0)^2}{R_0} = \frac{(kX)^2}{R_0} = xkX$$

Loading power factor:

$$p = \frac{R}{X} = kx$$

Fig. 20—Theorem for the loading power factor of a resonator coupled to a waveguide; proof by equivalent network.

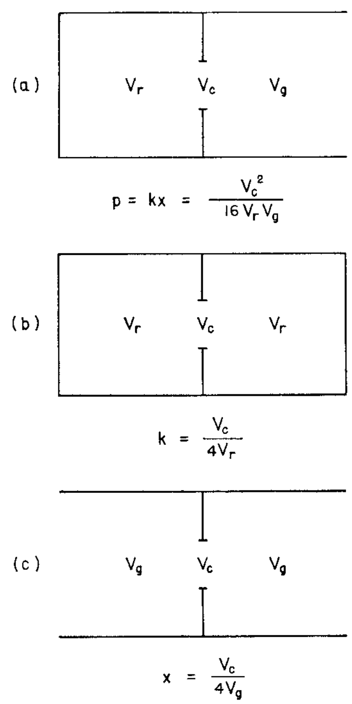
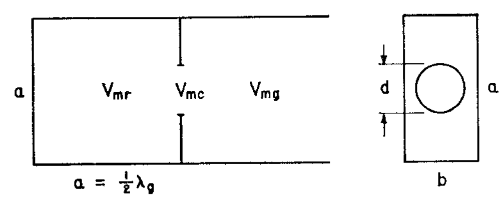


Fig. 21—A resonant cavity coupled to a waveguide (end wall).



$$V_{mr} = \frac{1}{2}a^2b; \qquad V_{mc} = \frac{2}{3}d^3; \qquad V_{mg} = \frac{1}{2\pi}a^2b$$

$$p_m = \frac{V_{mc}^2}{16V_{mr}V_{mg}} = \frac{\pi}{9}\left(\frac{d^3}{a^2b}\right)^2 = \frac{\pi}{144}\left(\frac{d}{b}\right)^6\left(\frac{2b}{a}\right)^4$$

As drawn: $b/a = 1/2$; $d/b = 2/3$; $p_m = 0.0019$

Fig. 22—Example of coupling between a resonant cavity and a waveguide.

tween a square cavity and a waveguide in order to simplify the formula and to emphasize the more interesting relations.

IX. Conclusion

A basis has been presented for evaluating the coupling through an aperture in terms of ratios of effective area or volume of the aperture relative to the adjoining bounded regions. The measure of coupling is the coupling coefficient between two resonant cavities, the normalized reactance or susceptance between two waveguides, or the loading power factor of coupling between a cavity and a waveguide. A simple theorem is presented for evaluating the unsymmetric last form from the viewpoint of the two symmetric forms.

In each case, there is a significant "power law" that shows the proportionality between the index of coupling and the diameter of the coupling hole. In the symmetric cases, the coupling (k or x) shows area or square-law proportionality for a two-dimensional field at the aperture, and shows volume or cube-law proportionality for a three-dimensional field at the aperture. In the unsymmetric case, the loading power factor is proportional to twice as high a power, namely, (area)^2 or fourth power, and (volume)^2 or sixth power.

The concepts and formulas presented here are intended as an aid in understanding and computing the behavior of coupling apertures between bounded regions of a wave medium.

Acknowledgment

The ideas presented herein have evolved over the years of association with the staff of Wheeler Laboratories in creative work on microwave components. It is a pleasure to acknowledge the writer's indebtedness to various members of the staff, and especially to Ned A. Spencer and David S. Lerner for helpful suggestions toward this presentation, to Henry J. Newton for drawing the illustrations, and to Mrs. Nancy M. Staron for typing the mathematics and text.

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